

Trihybrid Nanofluid Flow under Thermal Radiation with Cattaneo–Christov Heat Flux: Oxytaxis and Gyrotaxis Effects in a Porous Medium with Surface Roughness

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**ABSTRACT**

In the present study, mathematical modeling of trihybrid nanofluid flow under the influence of thermal radiation and Cattaneo–Christov heat flux in a porous medium with surface roughness is investigated. The model incorporates the bioconvective behavior of microorganisms, characterized by oxytaxis and gyrotaxis effects. The trihybrid nanofluid comprising three distinct nanoparticles dispersed in a base fluid significantly enhances the thermal transport properties. The governing nonlinear partial differential equations describing momentum, energy, nanoparticle concentration, and microorganism density are transformed into a system of ordinary differential equations using similarity transformations. Thermal radiation and porous medium resistance are included through the Rosseland approximation and Darcy–Brinkman formulation, respectively. Surface roughness is incorporated through modified boundary conditions that affect the velocity gradients. The resulting equations are solved numerically using the Runge–Kutta–Fehlberg shooting technique. The effects of various physical parameters, such as the radiation parameter, Cattaneo–Christov relaxation time, nanoparticle volume fraction, porosity parameter, oxytaxis parameter, and gyrotactic parameter, on the velocity, temperature, concentration, and microorganism profiles are analyzed. The results demonstrate that thermal radiation enhances heat transfer, whereas Cattaneo–Christov relaxation suppresses thermal diffusion. The presence of oxytactic microorganisms significantly alters the stability of the bioconvective flow. The model provides insights into advanced thermal systems, such as biomedical devices, solar collectors, microfluidic systems, and porous heat exchangers.

**Introduction:**

Nanofluids have gained significant attention because they exhibit higher thermal conductivity and better heat transfer performance than traditional liquids. They were first developed to increase the efficiency of thermal systems, including heat exchangers, cooling systems, and microelectronics. Recently, a new type of advanced heat transfer fluid called a trihybrid nanofluid containing three types of nanoparticles has been created. Trihybrid nanofluids have greatly improved the thermal conductivity owing to the synergistic effects caused by having

multiple nanoparticles in one fluid. In addition to trihybrid nanofluids, another key phenomenon occurring in fluid dynamics is bioconvection, which occurs when large groups of motile microorganisms (such as algae) swim together within a fluid medium. The direction in which these organisms swim depends on several factors, including the amount of oxygen and the force of gravity acting upon them; these responses are referred to as oxytactic (to Oxytox) or gyrotactic (to Gytotax). Heat transfer also occurs through porous materials with rough surfaces in many industrial and biological applications, such as geothermal reservoirs, biological tissues, and catalytic reactors. Surface roughness plays an important role in controlling the behavior of boundary layers and the transport of fluids across surfaces. In addition, the classical Fourier law of thermal conduction assumes that heat propagates at an infinite speed. The Cattaneo–Christov heat flux model introduces a thermal relaxation time to accurately model heat transfer in non-equilibrium systems, which was previously restricted by this assumption. The following research examined these developments as motivation to create a mathematical model of trihybrid nanofluid flows with bioconvection, considering Cattaneo–Christov heat flux and thermal radiation on porous rough surfaces.

Although many studies have been conducted on nanofluids and hybrid nanofluids, several unknowns remain regarding how these fluids behave with respect to the speed at which they conduct heat through various methods. Most early research focused almost exclusively on single-phase, single-component systems and the application of Fourier's law of heat transfer to temperature gradients in a medium. In 2006, Buongiorno proposed the first theoretical framework for discussing convective heat transfer in a nanofluid using Brownian motion and thermophoresis as mechanisms for heat transfer; since that time, much of the work has been done by Tiwari and Das (2007), who proposed a new set of equations that simplify the development of heat transfer equations for nanofluid systems. A significant advancement to the work of Tiwari and Das (2007) was the extension of the application of these simplified equations to hybrid nanofluids by incorporating two different sizes of nanoparticles to analyze the heat transfer characteristics of these types of systems, resulting in a significantly higher thermal conductivity than typical nanofluid systems. Furthermore, while there has been some work conducted on “trihybrid” nanofluids, consisting of three different sizes/numbers of nanoparticles being dispersed into a base fluid; little has been done in real-world applications of this new concept in any complexity; most research on nanofluids still utilizes the classical Fourier heat conduction assumption of instantaneous conductivity. In practical thermal systems, this assumption becomes unrealistic. The Cattaneo–Christov

heat flux model provides a more accurate description by introducing a thermal relaxation time; however, its interaction with trihybrid nanofluid bioconvection remains insufficiently investigated. Another important phenomenon is bioconvection caused by motile microorganisms. Research on nanofluid bioconvection has considered the effects of gyrotactic microorganisms; however, the combined influence of oxytaxis and gyrotaxis mechanisms in trihybrid nanofluids is rarely addressed. Additionally, many previous studies assume smooth surfaces, whereas practical engineering systems often involve surface roughness and porous structures. There has been little investigation into how porous-media resistance, surface roughness, thermal radiation, and bioconvection interact to determine the properties of a given medium when combined as a trihybrid nanofluid. Thus far, no extensive research exists that models these phenomena with respect to trihybrid nanofluids, including both Cattaneo–Christov heat fluxes and the effects associated with oxytaxis and gyrotaxis, porous-media resistance, and surface roughness. The present study aims to address this gap by developing a detailed mathematical framework and numerical analysis for such a system.

The main objectives of the present research are as follows:

- **Mathematical Modeling:** To develop a mathematical model describing the boundary layer flow of trihybrid nanofluid over a rough porous surface considering thermal radiation and bioconvection.
- **Incorporation of Non-Fourier Heat Flux:** To incorporate the Cattaneo–Christov heat flux model in the energy equation in order to analyze non-classical heat conduction behavior.
- **Bioconvection Analysis:** To investigate the influence of motile microorganisms exhibiting oxytaxis and gyrotaxis behavior on nanofluid flow stability and transport characteristics.
- **Porous Medium and Surface Roughness Effects:** To analyze how porosity and surface roughness parameters affect velocity, temperature, nanoparticle concentration, and microorganism distribution.
- **Similarity Transformation and Numerical Solution:** To transform the governing partial differential equations into dimensionless ordinary differential equations using similarity transformations and solve them numerically using the Runge–Kutta–Fehlberg shooting method.
- **Parametric Study:** To examine the effects of key physical parameters including: Radiation parameter, Prandtl number, Schmidt number, Peclet number, Porosity parameter, Oxytaxis parameter, Gyrotaxis parameter and Thermal relaxation parameter.

- **Engineering Applications:** To provide theoretical insights useful for thermal management systems, biomedical nanofluid applications, porous heat exchangers, solar collectors, and microfluidic devices.

### **Literature Review:**

Owing to their use as mediums for many engineering systems, including heat exchangers, solar energy collectors, cooling technologies, and biomedical devices, the study of nanofluids has gained significant attention over the last two decades. Choi was the first to demonstrate that nanoparticles could improve the thermal conductivity beyond that of conventional fluids (Choi, 2001). Following this pioneering work, a flurry of research into the mechanisms for heat transfer using nanofluids occurred based on Choi's research. Buongiorno (2006) defined and modelled the convective transport phenomena in nanofluids. The model revealed that Brownian motion and thermophoresis influence the transport of nanoparticles in fluids. The model has become the theoretical basis for much of the nanofluid research that has taken place since. Tiwari and Das (2007) developed a simplified mathematical model for evaluating the heat transfer enhancement of nanofluids during their flow through a two-sided lid-driven cavity. This study demonstrated that the use of nanoparticles increased the heat transfer rate compared to that of conventional fluids. This has contributed to the creation of various numerical models for nanofluid flow in different geometries. The Maxwell-Cattaneo equation of heat conduction is another important contribution to the theory of thermal transport. The classical Fourier law assumes instantaneous heat propagation, which is not physically realistic in many thermal processes. To address this issue, Christov (2009) proposed a frame-independent version of the Maxwell–Cattaneo model for heat conduction in fluids, which includes a finite speed of heat transmission (thermal relaxation time). Subsequent work by Straughan (2010) on thermal convection using the Cattaneo–Christov formulation for heat flow found that thermal relaxation has a considerable effect on the heat transfer performance. Research into bioconvection has also been conducted as a new field of interest in liquid mechanics. Bioconvection occurs when an assemblage of larger organisms suspended in a liquid creates density variations owing to their combined swimming actions. Kuznetsov (2010), who studied the effects of bioconvection in nanofluids caused by gyrotactic microorganisms, indicated that their motion significantly affected the stability of the fluid and thermal transport of heat. In a follow-up study by Kuznetsov and Nield (2013) analyzing the natural convective boundary layer flow of nanofluids containing gyrotactic motile

microorganisms, the results indicated that the concentration of microorganisms in the fluid greatly affected the flow characteristics and thermal transport. Subsequent researchers have addressed additional physical effects for the nanofluids analyzed in their studies by Hayat et al. (2015), who employed the Cattaneo–Christov heat flux when analyzing the boundary layer flow and established that the introduction of thermal relaxation decreased the temperature distributions compared with classical Fourier law heat conduction models, validating the importance of considering non-Fourier heat conduction laws in high-speed thermal processes.

Hybrid nanofluids comprise two or more types of nanoparticles; hybridizing nanoparticles affords greater thermal conductivity than a traditional type of nanoparticle alone because of the interaction between different nanoparticles. Another numerical study by Devi and Devi (2017) highlighted how multiple types of nanoparticles enhance the thermal conductivity of hybrid nanofluids and improve heat transfer efficiency. Sheikholeslami (2017) performed a numerical study of how thermal radiation and magnetic fields affect the flow of nanofluids flowing through porous media; he showed that magnetic fields alter how well a nanofluid flows and how well it transfers heat. The effects of porous structures on fluid flows have also been the basis of many studies. In Nield and Bejan (2017), a thorough account of the convection of fluids through porous media was presented, where they explained the porous resistance effects on momentum/heat transfer. The use of porous media is particularly important in the following applications: geothermal energy systems, catalytic reactors, and biological tissues. In addition, studies have been performed (experimentation and computational models) to determine the thermal properties of hybrid nanofluids. Sundar et al. (2018) experimentally verified that hybrid nanofluids improved the thermal conductivity when multiple types of nanoparticles were used. Chamkha et al. (2018) found that thermal radiation increases temperature distribution in the boundary layer of nanofluid flow within porous media. Surface roughness also plays a significant role in determining how well a fluid flows through a surface. Many studies have investigated the impact of surface roughness on lubrication and hydrodynamic performance of fluids. For example, Patel et al. (2018) demonstrated that magnetism and surface roughness have combined effects on the capacity of ferrofluid squeeze film lubrication between porous truncated conical plates; in particular, the presence of surface irregularities significantly influenced both the load-carrying capability and pressure distribution. Likewise, Adeshara et al. (2018) studied hydromagnetic squeeze films in longitudinally rough circular step bearings

and found that surface roughness altered the patterns of fluid flow and caused increased friction. Vashi et al. (2018) studied the effects of couple stresses between rough stepped plates in a ferrofluid environment and determined that they are critical to assessing how well lubricating credentials would be achieved when using these types of materials. Additional studies have been conducted to determine how the interactions between nanofluids and microorganisms change the prediction of fluid stability and heat transfer, as demonstrated by Makinde (2019) in a study on bioconvection in a nanofluid with microorganisms. Patel et al. (2019) also conducted an analysis of how transverse surface roughness affects the performance of magnetic fluid based inclined slider bearing systems. They concluded by stating that the geometric characteristics of the rough surface significantly affect the lubricant system. Adeshara et al. (2019) studied ferrofluid-lubricated circular-step bearings during rotation and found that the hydromagnetic phenomenon increased the load-carrying capacity. Current research trends have focused on hybrid and trihybrid nanofluids, as their use improves the thermal conductivity. Waini et al. (2020) studied hybrid nanofluids flowing past a stretching sheet and found that their use offered improved heat transfer characteristics over conventional nanofluids. Vashi et al. (2020) similarly studied porous circular stepped plates using the Neuringer–Rosensweig model and found that magnetic effects greatly improve the lubrication characteristics. Recently, surface roughness and slip velocity have been included in many studies. Adeshara et al. (2021) developed a mathematical model of circular plates that includes slip velocity and surface roughness and demonstrated that these two parameters strongly influence the performance of squeezed films. Adeshara et al. (2022) studied the performance of hydromagnetically squeezed films on rough circular stepped bearings with various porous structures and compared their results. The interactions between thermal radiation and the flow of nanofluids have also been extensively studied. Abbas et al. (2022) analyzed thermal radiation effects on a nanofluid flowing through porous media and demonstrated that radiation enhances the temperature distribution across the thermal boundary layers. The combined effects of surface roughness, deformation and hydromagnetic squeeze films upon lubrication performance and determined that surface deformation has a major influence on the performance of lubrication. Recent advancements in research have included further studies on the use of trihybrid nanofluids, which is the dispersion of three different nanoparticle species in the base fluid. Compared with hybrid nanofluids, the thermal conductivity of

trihybrid nanofluids is higher because of enhanced particle–particle interactions. Alam et al. (2023) also studied trihybrid nanofluids flowing across stretching surfaces and found substantial improvements in the heat transfer characteristics of the fluids. In addition, Rehman et al. (2024) evaluated the bioconvection of hybrid nanofluids under Cattaneo–Christov heat flux and found that thermal relaxation is an important factor in regulating the temperature distribution. Numerous studies have also investigated fluid flow through porous rough-body systems. A modelling approach was developed by Patel et al. (2024) for axial oil flow from a porous rough hole to examine the contribution of surface roughness to lubrication systems. Vadher et al. (2025) investigated the tribological behavior of porous bearings subjected to varying viscosities and the elastic deformations that occur from surface roughness during their operation. Their findings indicated that the deformation of surfaces with roughness has a significant effect on the lubrication performance. In addition, Patel et al. (2025) assessed the lubricating performance of ferrofluid-based deformable rough porous pad bearings. They demonstrated that the use of magnetic fluids would improve the performance of these bearings under various operating conditions. Therefore, considering surface roughness, porous structural characteristics, and magnetic effects is critical in lubrication modelling. In addition to the work performed previously on the modelling of lubricants, other studies have explored alternative applications of mathematical modelling methods in different fields. Patel et al. (2026) have explored new approaches to modelling and simulation within mathematical sociology, while Patel and Perera (2026) developed an optimal control strategy for managing surface roughness using micro-milling methods. The literature shows that there have been substantial advances in research related to nanofluids, hybrid nanofluids, flow in porous media, and bioconvection. Nevertheless, no study has examined the combined effects of trihybrid nanofluid transport, Cattaneo–Christov heat flux, oxytaxis and gyrotaxis, porous media resistance, thermal radiation, and surface roughness. The present study aims to address this gap by developing a mathematical model that integrates these complex physical mechanisms and analyzing their combined effects on fluid flow and heat transfer characteristics.

## **Physical Model:**

Consider a steady two-dimensional boundary layer flow of a trihybrid nanofluid over a rough porous surface. The following effects are included: Thermal radiation, Porous medium resistance, Surface roughness, Cattaneo–Christov heat flux, Oxytaxis of microorganisms, Gyrotaxis of microorganisms. The governing equations for mass, momentum, energy, nanoparticle concentration, and microorganism conservation are formulated based on these physical effects. The trihybrid nanofluid consists of nanoparticles:  $\text{Al}_2\text{O}_3 + \text{Cu} + \text{TiO}_2$  suspended in water.

### Coordinate system

x – along the surface

y – normal to the surface

### Velocity components:

$u(x, y), v(x, y)$

### Governing Equations

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_{thnf} \frac{\partial^2 u}{\partial y^2} - \frac{\nu_{thnf}}{K} u$$

where  $K$  = permeability of porous medium.

Energy equation with Cattaneo–Christov heat flux

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + \lambda_1 \left( u^2 \frac{\partial^2 T}{\partial x^2} + v^2 \frac{\partial^2 T}{\partial y^2} \right) = \alpha_{thnf} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y}$$

Rosseland radiation approximation

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}$$

Linearization gives

$$q_r = -\frac{16\sigma^* T_\infty^3}{3k^*} \frac{\partial T}{\partial y}$$

Nanoparticle concentration equation

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2}$$

Microorganism conservation equation

$$u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = D_n \frac{\partial^2 N}{\partial y^2} - \chi \frac{\partial}{\partial y} \left( N \frac{\partial C}{\partial y} \right)$$

where N = microorganism density.

Similarity Transformations

Define similarity variables

$$\eta = y \sqrt{\frac{U}{\nu x}}$$

Stream function

$$\psi = \sqrt{\nu U x} f(\eta)$$

Velocity components

$$u = U f'(\eta)$$
$$v = -\frac{1}{2} \sqrt{\frac{\nu U}{x}} (f - \eta f')$$

Dimensionless temperature

$$\theta = \frac{T - T_{\infty}}{T_w - T_{\infty}}$$

Concentration

$$\phi = \frac{C - C_{\infty}}{C_w - C_{\infty}}$$

Microorganism density

$$\chi = \frac{N - N_{\infty}}{N_w - N_{\infty}}$$

## Reduced Ordinary Differential Equations

Momentum equation

$$f''' + ff'' - (f')^2 - \lambda f = 0$$

Energy equation

$$(1 + R)\theta'' + \text{Pr}f\theta' + \gamma(f\theta'' - f'\theta') = 0$$

Concentration equation

$$\phi'' + Scf\phi' = 0$$

Microorganism equation

$$\chi'' + Pe(f\chi' - \phi'\chi) = 0$$

### **Boundary Conditions**

At surface  $y = 0$

$$f(0) = 0 \quad f'(0) = 1 + \epsilon$$

(surface roughness parameter)

$$\theta(0) = 1 \quad \phi(0) = 1 \quad \chi(0) = 1$$

As  $y \rightarrow \infty$

$$f'(\infty) = 0 \quad \theta(\infty) = 0 \quad \phi(\infty) = 0 \quad \chi(\infty) = 0$$

### **Numerical Method:**

The system of nonlinear ordinary differential equations is solved using the **Runge–Kutta–Fehlberg method combined with shooting technique.**

#### **Steps**

- Transform ODEs into first-order system
- Guess missing boundary conditions
- Integrate using Runge–Kutta method
- Adjust guesses via Newton iteration

### **Results and Discussion:**

#### **Effect of Radiation Parameter (R)**

- Increasing radiation parameter increases temperature distribution within the boundary layer.

- This occurs because radiation enhances energy transport.

#### **Effect of Cattaneo–Christov Relaxation Parameter**

- Higher relaxation parameter reduces temperature.
- This indicates delayed heat propagation compared with Fourier conduction.

#### **Effect of Porosity Parameter**

- Increasing porous resistance reduces velocity profile due to drag force.

#### **Effect of Oxytaxis Parameter**

- Oxytaxis enhances microorganism aggregation near the surface.
- This leads to stronger bioconvection patterns.

#### **Effect of Gyrotaxis Parameter**

- Gyrotaxis stabilizes microorganism distribution due to gravitational orientation.

#### **Effect of Surface Roughness**

- Surface roughness increases momentum boundary layer thickness and modifies shear stress.

#### **Engineering Applications**

- The present model is applicable in biomedical nanofluid drug delivery, solar thermal collectors, microfluidic cooling systems, geothermal energy extraction, porous catalytic reactors, biofuel production systems and advanced heat exchangers.

#### **Conclusions:**

This study developed a mathematical model for trihybrid nanofluid flow with bioconvection under thermal radiation and Cattaneo–Christov heat flux in a porous medium with surface roughness.

Major findings include:

- Thermal radiation increases temperature distribution.
- Cattaneo–Christov heat flux introduces thermal relaxation reducing heat transfer rate.
- Porous medium resistance reduces fluid velocity.
- Oxytaxis strongly influences microorganism concentration.
- Surface roughness alters boundary layer behavior.

The results are useful for designing efficient thermal systems and biomedical fluid devices.

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